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# Children's Strategies in Approximate Quantification

Delphine Gandini, Eléonore Ardiale and Patrick Lemaire

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- 1 The present study investigated how fifth and seventh graders process approximate quantification task. Approximate quantification refers to our ability to provide a quick and rough estimate of a magnitude. Understanding this ability is important. First, everyday, we are bombarded by a lot of numerical information. Due to environmental constraints (e.g., time pressure), we often process this numerical information in an approximate manner. For instance, in a supermarket, to choose the checkout where we shall wait for, we generally approximate the number of persons waiting for each checkout and choose the one with the fewest persons. Similar processes are involved when we want to determine if the available place to park our car is sufficiently large or if the quantity of food in our plate is reasonable. So studying this ability and the effect of age on it is important both because of its pervasiveness and because of its centrality for understanding our social and physical environment.
- 2 Few studies have documented effects of age on approximate quantification (e.g., Ansari, Donlan, & Karmiloff-Smith, 2007; Booth & Siegler, 2006; Crites, 1992; Luwel, Verschaffel, Onghena, & De Corte, 2003a, b; Luwel, Lemaire, & Verschaffel, 2005; Opfer & Siegler, 2007; Siegel, Goldsmith, & Madson, 1982; Siegler & Booth, 2004; Siegler & Opfer, 2003). Nevertheless, all of these studies showed increasing accuracy of estimates with age. This increased accuracy of estimates with age is in part the result of age-related changes in memory representations of numerosities and strategies that are used to find estimates.
- 3 This finding is consistent with overlapping waves model (Siegler, 1996). According to this model, cognitive development is characterized by two fundamental facts: the coexistence of different procedures to accomplish a task and a constant change in the frequency use of these procedures. This variability stems from maturational changes (internal or cognitive factors) and problems' characteristics (external or situational factors). According to Lemaire and Siegler (1995), different cognitive components must be taken into account to better describe participants' performance and its variability,

the so-called strategy repertoire, distribution, execution, and selection. These different dimensions refer to the different strategies used by an individual to accomplish a task, the performance resulting from the utilisation of a particular strategy, and the variables influencing how participants choose among strategies. Shrager and Siegler (1998) proposed a model to explain the mechanisms of strategy selection and how the different strategy dimensions interacted to produce adaptive strategy choices. This model includes among others associative-learning mechanisms. As a child solves a problem, the problem and answers stated on this problem are associated. The strengths of alternative strategies for solving such problems are continually adjusted in accord with the relative speeds and accuracies they produce on the problems. As a consequence, with the development, an increase in performance and use of more sophisticated and adaptive strategy occurs. This framework points to the importance to take into account the interaction between the individual and environmental characteristics to better understand the development.

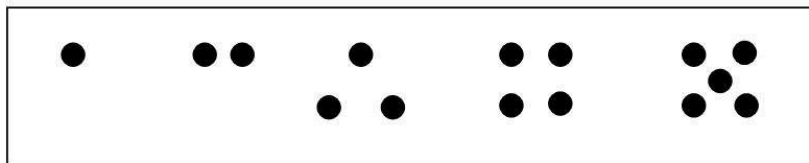
- 4 Concerning the specific domain of quantification, previous works suggest that both adults and children use different strategies (e.g., Camos, 2003; Crites, 1992; Luwel et al., 2003a, b, 2005; Siegel et al., 1982). For instance, Luwel and colleagues showed that children of second and sixth graders, and young adults used three main strategies to make numerosity judgements of coloured blocks in grids comprising 10x10 blocks: (a) the addition strategy consisting in dividing the given quantity of blocks into a number of subgroups and adding the judged numerosities of the different subgroups, (b) the subtraction strategy consisting in determining the number of empty blocks (by means of an addition strategy) and subtracting this from the total number of blocks in the grid, and (c) the estimation strategy, corresponding to a quick and rough process of quantification (i.e., guessing). Camos (2003) specified the addition strategies use on small numerosities (i.e., 8 to 25) by 5- to 15-years old and adults, showing diversity in the addition processes for exact quantification. Moreover, these authors showed that strategy use and strategy execution varied with participants' and items' characteristics. Similar variability has been found with ecological estimation problems (e.g., children saw a whisk and its accompanying question "About how many ridges are there on the handle of this whisk?"), showing that children tended to use more sophisticated strategies with development (Crites, 1992; Siegel et al., 1982).
- 5 Although previous studies reported strategy variations while children and young adults accomplish quantification tasks, we do not know precisely what are the different strategies available to make approximate quantification. Indeed, even if Luwel and colleagues clearly identified an estimation strategy for judging numerosity of blocks, we do not know the set of processes this estimation strategy includes and whether there is one versus several types of estimation strategies. One important goal of the present study was to investigate in great details the strategies that participants use to solve visual approximate quantification tasks, and how participants select among strategies on each item.
- 6 Thus, the originality of this paper stems from two aspects, one conceptual and one empirical. First, from a conceptual point of view, we adopted a strategy perspective. From an empirical point of view, we collected different behavioural measures, such as trial-by-trial strategy reports, accuracy, and latency. This enabled us to assess the effects of participants' age on approximate quantification performance, strategy repertoire, and strategy selection as a function of item characteristics in each age

group. Despite their limits regarding reactivity and validity (see Ericsson & Simon, 1993; Kirk & Ashcraft, 2001; Robinson, 2001; Smith-Chant & LeFevre, 2003), immediate retrospective verbal protocols provide fruitful data to understand participants' strategies. Previous studies showed that participants are able to describe their strategies in a fairly accurate and reliable way, especially in numerical processing tasks (see Lemaire, 2005, for a discussion).

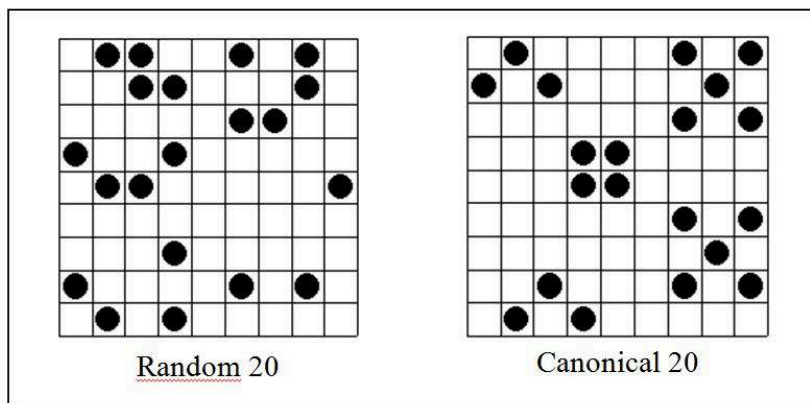
- 7 In this study, fifth and seventh graders were asked to estimate numerosities of collections of 11–79 dots. In addition to the number of dots, we manipulated their configuration (random vs. canonical; see Figure 1). We manipulated configurations of dots because previous studies showed reliable effects of spatial layouts of elements on participants' performance (e.g., Bevan, Maier, & Helson, 1963; Frith & Frith, 1972; Ginsburg, 1991; Krueger, 1972; Vos, van Oeffelen, Tibosch, & Allik, 1988).

Figure 1: (a) The five canonical patterns used in Experiments 1 and 2. (b) Examples of canonical and random items with 20 dots.

(a)



(b)



- 8 This experiment enabled to test the strategy variability hypothesis as well as age-related differences in approximate quantification skills and in cognitive strategies. According to the strategy variability hypothesis, (a) participants use several strategies, and (b) strategy use vary as a function of item characteristics. Moreover, age-related differences in strategic variations predict that fifth and seventh graders differ (a) in the strategy repertoire used to accomplish approximate quantification tasks, (b) how often they use each available strategy, and (c) how problem features affect strategy distributions.

## Method

- 9 Participants. Forty-nine children participated: 25 fifth graders (mean age= 10.4 years,  $SD= 0.4$ ) and 24 seventh graders (mean age= 12.8 years,  $SD= 0.5$ ). Children were recruited in two different public schools from Marseille and Aix-en-Provence.
- 10 Stimuli. The stimuli were 108 configurations of 8-mm black dots displayed in a visible square grid on a white background, two-thirds of which were experimental stimuli (including 15, 20, or 25 dots) and one-third of which were fillers (including 11–79 dots, excluding collections of 15, 20, or 25 dots). The set of 108 grids was divided into two sets of 54 trials each. Each grid was made of 100 (10 x 10) units; each unit had a size of 1 x 1 cm square (participants sat 60 cm from the screen, meaning that each grid occupied 9.5 degrees of visual angle). The minimum distance between dots was 4 mm, and between dots and grid 2 mm. Six types of grids were tested on the basis of the size of the numerosity and configuration of black dots. Based on previous findings showing strategic variations in adults with numerosities (e.g., Gandini, Lemaire, & Dufau, 2008b; Luwel et al., 2003a & b; 2005), the seventy-two experimental stimuli always had 15, 20, or 25 dots<sup>1</sup>. In a given grid, the arrangement of dots was either random or canonical (i.e., being composed of canonical patterns; see Figure 1). These experimental trials were intermixed with 36 fillers containing between 11 and 79 randomly displayed dots. In each block of 54 trials, 18 had a random configuration, 18 had a canonical configuration, and 18 were fillers.
- 11 Canonical configurations were constructed with several constraints: (a) We controlled the mean number of groups of (1–5) dots composing an item, independently of the pattern types: On average, displays always contained 5, 7, and 9 groups of dots for the numerosities 15, 20, and 25, respectively; (b) Displays were matched on the number of different pattern types appearing on a grid: Canonical displays were always combinations of three types of canonical patterns (e.g., a canonical display could contain two patterns of five dots, two patterns of two dots, and one single dot); (c) We controlled the percentages of each pattern type: On average, each pattern type appeared on 20% of all canonical trials, except for the patterns of one single dot and five dots, which appeared respectively on 18% and 22% of all canonical items. These distributions were identical for the three target numerosities; (d) For each target numerosity, we controlled that each pattern appeared on the same number of grids; (e) Following Mandler and Shebo (1982), no shapes other than those in Figure 1a were used for groups of dots (e.g., triangle of three dots always had its apex at the top; four dots could not form a diamond).
- 12 Procedure. Children were tested individually in one session that lasted approximately 40 minutes. Stimuli were presented using a DELL Latitude 120 L computer with a 14-inch computer screen. The experiment was controlled by the E-Prime software. The program generated the displays and recorded latencies to the nearest millisecond. The display resolution was 640 x 480 pixels. Each trial was preceded by a blank screen (1000 ms) and a fixation point ("\*") in the center of the screen for 750 ms. The grid was then displayed in the center of the screen during 6 seconds. Children were instructed to provide their estimates within 6 seconds. The grid remained on the screen until the participant responded or 6 s elapsed (which happened on 14% of all trials). If an estimate was not provided after 6 s, the grid disappeared and four blue crosses were displayed. After each grid, the experimenter recorded participant's response and

verbal protocol (i.e., children were asked “how did you estimate the number of dots?”). Verbal protocols were fully written down by the experimenter for later coding. Instructions mentioned no particular strategies. Grids did not remain on the screen during verbal protocols. A timer was started when the grid appeared on the screen and ended when the experimenter pressed on the space bar of the computer keyboard, which happened as soon as possible after children provided their response orally<sup>2</sup>. The order of presentation of grids was randomized for each participant.

- 13 General instructions described the approximate quantification task (i.e., “providing approximate number of dots displayed on the square grid without counting the exact number”), and children were asked to respond as quickly as possible but without sacrificing accuracy. Each child was permitted a 5-10 minute rest between blocks. Before the experiment starts in earnest, children received 12 practice (similar to but different from) experimental trials to familiarize themselves with the apparatus, procedure, and task.

## Results

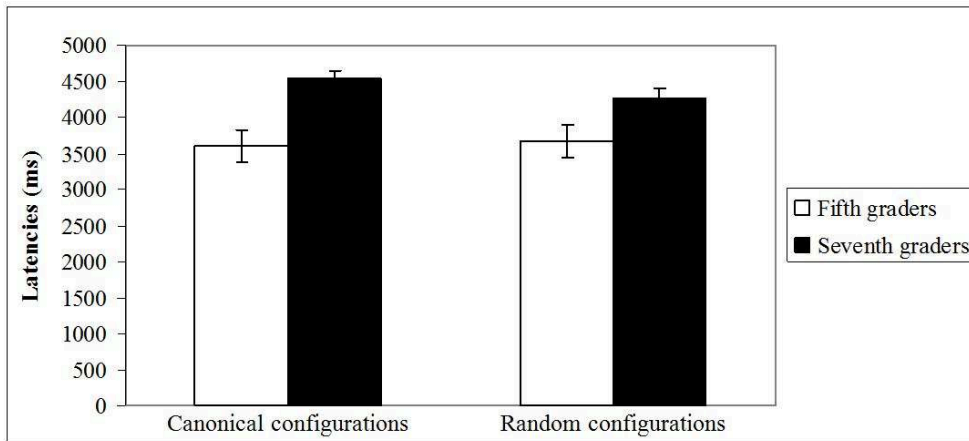
- 14 Results are reported in three main parts. The first examines age-related differences in approximate quantification performance (i.e., RTs and percentages of deviation). The second looks at which strategies participants used and strategy frequencies. Finally, the third examines age-related differences in strategy selection. In all results, unless otherwise noted, differences are significant to at least  $p < .05$ .

### Approximate quantification performance

- 15 Following previous works on estimation (e.g., Gandini et al., 2008b; Gandini, Lemaire, Anton, & Nazarian, 2008a), to measure estimation accuracy, we calculated each participant's percent absolute deviation:  $\frac{[(\text{Estimate} - \text{Exact numerosity})/\text{Exact numerosity}] * 100$ . To illustrate, suppose a child gave 20 as an estimate for 25 dots. That child would be 20%  $\frac{[(20-25)/25] * 100$  away from the exact numerosity.
- 16 Mean RTs and percentages of deviation were analyzed with ANOVAs with a 2(age: fifth, seventh graders) x 2(configuration: random, canonical) x 3(numerosity: 15, 20, 25) design, with age as the only between-subjects factor<sup>3</sup>.
- 17 ANOVA on RTs revealed a significant main effect of age,  $F(1,43)=11.8$ ,  $MSe=3352826$ , showing that seventh graders (4404 ms) were slower than fifth graders (3637 ms). As can be seen from Figure 2, the Age x Configuration interaction was significant,  $F(1,43)=12.08$ ,  $MSe=154981$ , showing that the age-related difference was larger on canonical (+934 ms,  $F(1,43)=17.21$ ,  $MSe=1704082$ ) than on random configurations (+600 ms,  $F(1,43)=6.72$ ,  $MSe=1803725$ ). Finally, the Age x Configuration x Numerosity interaction was also significant,  $F(2,86)=6.24$ ,  $MSe=82382$ . To further analyze this interaction, separate ANOVAs involving 2(configuration: canonical, random) x 3(numerosity: 15, 20, 25) designs were conducted in each age group. As can be seen in Table 1, the canonical-random differences varied with numerosities in seventh graders (-289 ms, -445 ms, and -37 ms, for 15, 20, and 25, respectively), as revealed by the significant Configuration x Numerosity interaction,  $F(2,46)=6.89$ ,  $MSe=73726$ . This interaction was not significant in fifth graders,  $F < 1.62$ . Moreover, main effects of

configuration,  $F(1,23)=15.8$ ,  $MSe=150919$ , and numerosity,  $F(1,46)=4.21$ ,  $MSe=105038$ , were found in seventh but not in fifth graders,  $F_s<1.16$ .

**Figure 2: The Age x Configuration Interaction on the Mean Latencies.**



- 18 The corresponding analyses of mean percentages of deviation revealed significant main effects of age,  $F(1,43)=17.82$ ,  $MSe=142.04$ , configuration,  $F(1,43)=14.07$ ,  $MSe=83.98$ , and numerosity,  $F(2,86)=14.71$ ,  $MSe=50.02$ . Seventh graders (18.7%) were more accurate than fifth graders (24.9%). Children produced better estimates on canonical (19.7%) than on random displays (23.9%), and their mean percentages deviation increased with numerosity (15: 18.7% vs. 20: 22.3% vs. 25: 24.3%, all pairwise comparisons were significant,  $F_s>4.40$ ). Moreover, all interactions were significant: Age x Configuration,  $F(1,43)=27.21$ ,  $MSe=83.98$ , Age x Numerosity,  $F(2,86)=9.09$ ,  $MSe=50.02$ , and Configuration x Numerosity,  $F(2,86)=130.01$ ,  $MSe=37$ . These interactions were further qualified by an Age x Configuration x Numerosity three-way interaction,  $F(2,86)=11.63$ ,  $MSe=37$ . Separate ANOVAs involving 2(configuration: canonical, random) x 3(numerosity: 15, 20, 25) designs were conducted in each age group. As can be seen in Table 1, the canonical-random differences decreased with numerosities in seventh graders and changed of direction (-22.8%, +10%, and +7.9%, for 15, 20, and 25, respectively; all pairwise comparisons were significant,  $F_s>31.34$ ), as revealed by the significant Configuration x Numerosity interaction,  $F(2,46)=177.43$ ,  $MSe=22.83$ . These differences also decreased in fifth graders (-2%, +20.7%, and +11.3%, for 15, 20, and 25, respectively),  $F(2,40)=25.67$ ,  $MSe=53.29$ , but were only significant for 20 and 25 dots,  $F_s>45.69$ .

**Table 1: Mean Latencies and Percentages of Deviation in Fifth and Seventh Graders, as a Function of Configuration and Numerosity.**

	Canonical configurations			Random configurations			Means			Overall
Numerosities	Fifth graders	Seventh graders	Means	Fifth graders	Seventh graders	Means	Fifth graders	Seventh graders	Means	
	<b>Latencies</b>									
15	3663	4476	4069	3658	4186	3922	3661	4331	3996	

20	3503	4592	4047	3716	4146	3931	3610	4369	3989
25	3630	4532	4081	3652	4494	4073	3641	4513	4077
Means	3599	4533	4066	3675	4276	3975	3637	4405	4021
Percentages of deviation									
15	20,7	29,1	24,9	18,7	6,3	12,5	19,7	17,7	18,7
20	14,7	14,6	14,6	35,4	24,6	30	25,1	19,6	22,3
25	24,2	14,9	19,5	35,5	22,8	29,2	29,9	18,9	24,3
Means	19,8	19,5	19,7	29,9	17,9	23,9	24,9	18,7	21,8

## Strategy use

- 19 Analyses of strategy use aimed at answering the following questions: (a) What strategies did participants use to provide estimates? (b) Did individuals use a single or several strategies? and (c) Which strategies were used most often? Two raters who independently classified strategy use agreed on 98% of them.
- 20 Analyses of individual protocols revealed six strategies (see examples of verbalizations in Table 2): (a) Anchoring: Children enumerated several dots (via counting), visually estimated the remaining dots based on the first enumeration, and then added the enumerated result and the estimated result, (b) Benchmark: Children visually scanned the stimulus, retrieved a numerical representation in long-term memory (LTM), compared the difference between the encoded representation and the retrieved representation, and then adjusted their answer on the basis of this difference, (c) Decomposition/Recomposition: Children spotted one group of few dots, up to about four or five items, estimated the number of analogous groups, and then multiplied the number of items primarily subitized by the estimated number of groups, (d) Approximate counting: Children perceived several groups of different sizes and approximately added these groups to produce estimates, (e) Exact counting: Children counted all dots displayed in the grids by systematically adding all items (by ones, twos, or threes), and (f) Others: These strategies included verbal reports that did not correspond to any of the previous categories.

**Table 2: Examples of Verbal Reports and Mean Percent use of Each Strategy, in Fifth and Seventh Graders.**

Strategies	% use in fifth graders	% use in seventh graders	Examples of verbal reports
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Anchoring	14	26	"I first counted 3 dots, then 4 dots, added 3 and 4=7. Then, I estimated that there remained approximately twice as many dots, so I figured that there are $7 + 14=21$ dots"
Benchmark	47	12	"I quickly looked at all the dots, thought it looked like there are about 30 or a little bit more, so I said 32"
Decomposition/ Recomposition	3	16	"I saw a group of 3 dots, and I estimated that there were six other similar groups; so I multiplied 7 by 3, and thought there are approximately 21 dots"
Approximate Counting	19	20	"I first saw two groups of 5 dots, which is 10 dots. Then I saw a group of about 6 dots, which is about 16 dots. Finally, I saw a group of 5 dots. So, there are approximately 21 dots"
Exact Counting	17	26	"I counted exactly each dot by three, 3, 6, 9, 12; there are 12 dots"
Other	0	0	"There are 100 blocks in a grid and I think there are approximately 80 empty blocks; So I found $100 - 80=20$ dots"

- 21 To determine whether children used a single or several strategies, we tallied the number of participants using zero, one, two, three, or more strategies. In fifth graders, four participants used only one strategy, six participants used two strategies, seven used three strategies, seven used four strategies, and one participant used five strategies. In seventh graders, one participant used two strategies, seven participants used three strategies, twelve used four strategies, and four used five strategies.
- 22 Moreover, to assess intra-individual variability, we analyzed the number of strategies used by participants across conditions. On average, seventh graders (3.8) used more strategies than fifth graders (2.8),  $F(1,47)=12.31$ ,  $MSe=0.98$ . The mean number of strategies used in each of the experimental conditions was also computed for each participant and analyzed with an ANOVA with a  $2(\text{age}) \times 6(\text{experimental condition})$  design, with age as the only between-subjects factor. The number of strategies per condition tended to increase with age from 1.4 at 10 years to 1.6 at 12 years ( $F(1,43)=3.13$ ,  $p<.08$ ).
- 23 Inter-individual variability was also assessed by computing the overall number of different strategies observed in each age group across conditions. The two age groups used the 5 strategies on the different types of configurations. However, an ANOVA with the 6 experimental conditions as a random factor and age as within-subject factor on the number of different strategies observed in each condition showed a significant age-related increase,  $F(1,66)=43.23$ ,  $MSe=0.39$ . Ten-year-old children used on average 2.7 strategies per condition, whereas 12-year-old children used 3.4 strategies. So, intra- and inter-individual variations in strategies, across or per condition, were present.

- 24 Next, we computed the overall percentages of each strategy use, for each individual, to analyze strategy preference. An ANOVA involving a 2(Age: fifth, seventh graders) x 5(Strategy: benchmark, anchoring, decomposition/recomposition, approximate counting, exact counting) design, with age as the only between-subject factor, revealed that fifth and seventh graders did not prefer the same strategies,  $F(5,215)=9.25$ ,  $MSe=398.5$ . Fifth graders' most favourite strategies were benchmark (which participants used on 47% of all trials on average), followed by approximate counting (19%), exact counting (17%), anchoring (14%), and finally decomposition/recomposition (3%); they used the other strategies on less than 1% of all trials. Seventh graders did not really prefer one strategy, using equally anchoring (26%), exact counting (26%), approximate counting (20%), and decomposition/recomposition strategies (16%). Finally, they used benchmark on 12% of all trials and the other strategies on less than 1%.

## Strategy selection

- 25 A third series of analyses aimed at examining the role of stimulus characteristics in fifth and seventh graders' strategy choices<sup>4</sup>. Mean percentages of use of each strategy were analyzed separately, with ANOVAs involving 2(Age: fifth, seventh graders) x 2(Configuration: random, canonical) x 3(Numerosity: 15, 20, 25) designs, with age as the only between-subjects factor. As the other strategies were used too rarely, we focused on the other five strategies (see means in Table 3).

**Table 3: Mean Percent use of Each Strategy in Fifth and Seventh Graders, as a Function of Numerosity and Configuration.**

	Canonical configurations			Random configurations			Overall
Numerosities	Fifth graders	Seventh graders	Means	Fifth graders	Seventh graders	Means	Means
				Anchoring			
15	13	27	20	8	9	9	14
20	23	43	33	11	12	12	22
25	16	44	30	17	31	24	27
Means	17	38	28	12	17	15	21
				Benchmark			
15	51	12	32	39	7	23	27
20	47	16	32	41	6	24	28
25	56	19	38	46	12	29	33

Means	51	16	34	42	8	25	29
				Decomposition/Recomposition			
15	2	16	9	6	21	14	11
20	3	14	9	2	17	10	9
25	4	12	8	2	15	9	8
Means	3	14	9	3	18	11	10
				Approximate counting			
15	20	24	22	19	16	18	20
20	19	21	20	20	15	18	19
25	17	18	18	19	25	22	20
Means	18	21	20	19	19	19	19
				Exact counting			
15	15	20	18	28	47	38	28
20	7	6	7	27	50	39	23
25	8	6	7	15	17	16	12
Means	10	11	11	23	38	31	21

- 26 Benchmark strategy. Fifth graders (47%) used the benchmark strategy more often than seventh graders (12%),  $F(1,43)=18.62$ ,  $MSe=4318.6$ . Moreover, participants used it more often while quantifying canonical than random configurations (33% vs. 25%,  $F(1,43)=15.77$ ,  $MSe=285$ ), and used it more often on configurations of 25 dots (33%) than on configurations of 15 (27%) or 20 dots (27%; the two pairwise comparisons were significant,  $F_s > 10.68$ ).
- 27 Anchoring strategy. Seventh graders (26%) used the anchoring strategy more often than fifth graders (14%),  $F(1,43)=10.76$ ,  $MSe=1066$ . ANOVA also revealed main effects of configuration,  $F(1,43)=31.06$ ,  $MSe=376.2$ , and numerosity,  $F(2,86)=16.6$ ,  $MSe=216.4$ , showing that children used it more often on canonical (28%) than on random configurations (14%), and more often as the numerosity increased (15: 14% vs. 20: 22% vs. 25: 27%; all pairwise comparisons were significant,  $F_s > 4.85$ ). These two factors interacted with age, and between them, showing that: (a) seventh graders used

- significantly more often the anchoring strategy than fifth graders on canonical configurations (38% vs. 17%,  $F(1,43)=14.39$ ,  $MSe=1000.9$ ), but not on random configurations (17% vs. 12%,  $F<2.25$ ); (b) the age-related differences increased with increasing numerosities (these differences were 7%, 11%, and 21%, for 15, 20, and 25 dots, respectively,  $F_s>4.27$ ); and (c) the random-canonical differences varied with numerosities (these differences were 12%, 22%, and 6%, for 15, 20, and 25 dots, respectively; pairwise comparisons were significant for the numerosities of 15 and 20,  $F_s>21.78$ ).
- 28 **Decomposition/Recomposition strategy.** Seventh graders (16%) used decomposition/recomposition strategy more often than fifth graders (3%),  $F(1,43)=4.65$ ,  $MSe=2375.4$ . Moreover, children varied their strategy use with the size of collections,  $F(2,86)=3.57$ ,  $MSe=49.2$ : They used it more often to estimate 15 dots (11%) than 25 dots (8%,  $F(1,43)=7.78$ ,  $MSe=43.1$ ).
- 29 **Approximate counting strategy.** ANOVA only revealed an interaction between configuration and numerosity,  $F(2,86)=3.73$ ,  $MSe=145.6$ , showing that participants used this strategy more often on canonical than on random configurations containing 15 dots,  $F(1,43)=4.13$ ,  $MSe=121.4$ .
- 30 **Exact counting strategy.** Children used the exact counting strategy more often on random (31%) than on canonical configurations (10%),  $F(1,43)=64.83$ ,  $MSe=424.9$ , and less often as the numerosity increased (15: 27% vs. 20: 22% vs. 25: 12%, all pairwise comparisons were significant,  $F_s>6.59$ ). Moreover, all interactions, Age x Configuration,  $F(1,43)=8.14$ ,  $MSe=424.9$ , Age x Numerosity,  $F(2,86)=4.38$ ,  $MSe=220.2$ , Configuration x Numerosity,  $F(2,86)=21.9$ ,  $MSe=140.6$ , and Age x Configuration x Numerosity,  $F(2,86)=4.21$ ,  $MSe=140.6$ , were significant, showing that: (a) the random-canonical differences varied with the size of numerosities (these differences were +20%, +32%, and +8%, for 15, 20, and 25 dots respectively,  $F_s>16.29$ ); and (b) seventh graders' strategy use was influenced by configuration and numerosity, as revealed by the significant Configuration x Numerosity interaction,  $F(2,46)=26.14$ ,  $MSe=130.2$  (the random-canonical differences were +27%, +44%, and +11%, for 15, 20, and 25 dots respectively,  $F_s>10.87$ ). This interaction was not significant in fifth graders,  $F<3$ .
- 31 **In summary,** children chose strategies on each item as a function of stimuli characteristics as shown by different strategy use varying with configurations and/or numerosities. In some cases, children took only numerosities into account (e.g., decomposition/recomposition strategy). In other cases, they were influenced by both configuration and numerosity (e.g., the benchmark, anchoring, approximate and exact counting strategies). Critically, the two age groups were not influenced in the same way by our two stimulus features for choosing the anchoring and exact counting strategies. Indeed, for these two strategies, 12-year-old children selected strategies on the bases of configuration and numerosity whereas 10-year-old children did not calibrate their strategy use on these parameters, using both strategies equally often on each type of configurations.

## Discussion

- 32 This study documented group differences in approximate quantification. A strategy perspective enabled us not only to compare fifth and seventh graders' performance,

but also to examine how children found approximate numerosities of collections of dots.

- 33 The first set of interesting findings in this experiment concerns performance. Like in previous studies of approximate quantification, speed and accuracy were influenced by problem features and participants' age. Children produced more accurate estimates on small than on large numerosities, and on canonical than on random items. Moreover, these two variables interacted between them and with age, showing that the canonical-random differences varied with the size of numerosities. Most likely, this arose because canonical patterns made it easy for participants to quickly encode collections of dots and retrieve a corresponding approximate numerosity from LTM (Mandler & Shebo, 1982). The extra-processing of parsing collections of dots so as to distinguish among them is more error-prone for the random than for the canonical patterns, and even more so when numerosities increase.
- 34 Regarding effects of age on solution times, surprisingly seventh graders were slower (but not less accurate) than fifth graders. Moreover, problem features only influenced seventh graders' latencies, with larger effects of configurations on large numerosities. These age-related differences may stem from strategy distributions. It is possible that participants, especially seventh graders, used more-time consuming strategies on random, large-numerosity items. Although this experiment did not enable to test this hypothesis directly via the analysis of execution components (as there were too many missing cells caused by strategy selection), the data are consistent with this explanation. Indeed, seventh graders preferred to use the exact counting strategy to approximate random configurations whereas fifth graders favored the benchmark strategy. Moreover, consistent with Gandini et al.'s (2008b) finding, children of both age groups took more time to estimate arrays of dots while using exact counting (4547 ms) than when they used benchmark (3796 ms,  $F(1,17)=16.50$ ,  $MSe=307674$ ).
- 35 The second set of interesting findings concerns strategy use. Like in arithmetic (e.g., Barrouillet, Mignon, & Thevenot, 2008; Lemaire & Lecacheur, 2002b), both age groups did not use a single strategy to approximate arrays of dots. Indeed, children used several strategies to accomplish approximate quantification task: benchmark, anchoring, decomposition/recomposition, approximate counting, exact counting, and other strategies. Some strategies were used more often (e.g., benchmark or approximate counting) than others (e.g., decomposition/recomposition or others). Strategy use was influenced by configurations, numerosities, and participants' characteristics.
- 36 Configuration of dots affected the use of anchoring, benchmark, and exact counting strategies, and numerosity influenced the use of all strategies, except approximate counting. The Configuration x Numerosity interaction was significant when children used anchoring, approximate and exact counting strategies, but not when they used benchmark or decomposition/recomposition. These different strategy distributions as a function of problem type show that children selected strategy on a problem-by-problem basis and adjusted their strategy use to both configurations and numerosities. Effects of configuration and numerosity on strategy use validate distinctions among strategies as the use of different strategies should be differently influenced by problem features.
- 37 Another interesting result concerns age-related differences in the mean number of strategies. First, although fifth and seventh graders' strategy repertoire included the

same set of strategies, fifth graders used fewer strategies than seventh graders, across and per condition. So, the increased number of strategies used by children may not come from strategy repertoire becoming larger with age. Rather, it is possible that, given reduced working-memory resources in fifth graders (see Hitch, 2006, for an overview), younger children focused on fewer strategies to choose among. Choosing among fewer strategies may feel easier than choosing among more strategies and may require fewer working-memory resources. Future studies, manipulating availability of working memory resources should tell us if reduced working-memory resources lead children to use fewer strategies to accomplish approximate quantification tasks.

- 38 Second, fifth and seventh graders differed in how often they used each available strategy. In particular, seventh graders used equally often anchoring, exact, and approximate counting, and decomposition/recomposition strategies, but less often the benchmark strategy. Fifth graders preferred benchmark and used it more often than anchoring, exact counting, or approximate counting strategies. This result is surprising as the benchmark strategy involves retrieving numerical representations in long-term memory whereas all the other strategies involve counting processes. Recall that Siegler and colleagues (Booth & Siegler, 2006; Siegler & Opfer, 2003; Opfer & Siegler, 2007) have found that younger children have less precise numerical representations than older ones. So, younger children were expected to use more often strategies based on counting, such as anchoring, approximate and exact counting, or decomposition/recomposition strategies. The inverse pattern of results would be expected for older children. One explanation of these results may stem in strategy characteristics. It is possible that seventh graders wanted to be most accurate, even if instructions did not particularly emphasize accuracy. They could achieve this end by using strategies based on counting more often than fifth graders. In contrast, fifth graders may have favoured speed and used benchmark more often.
- 39 Another non-exclusive explanation refers to children's general cognitive abilities, such as perception, attention, and memory. As suggested by Simon (1997), numerical abilities not only are based on numerical knowledge, but also require more general domain-independent competences. When the human cognitive system finds itself faced with a novel task, very general information processing resources are deployed for appropriately responding. Among these general cognitive resources, numerical competences involve memory and comparison of perceived entities, limited individuation and discrimination abilities (Trick & Pylyshyn, 1994), or abstract encoding process and numerical representations (Xu & Spelke, 2000). All of these abilities improve with development. So we can hypothesize that fifth graders preferred to use benchmark strategy because, given the 6-second time deadline in the present task, they were too limited in their general resources to use other strategies, such as anchoring, decomposition/recomposition or approximate counting strategies. Indeed, these strategies involve more complex sets of cognitive processes (Gandini et al., 2008b). Recall that, in these three strategies, participants were separating visual stimulus into different sub-collections of 3–5 dots (that they could subitize), and keeping track in visuo-spatial working memory the distinction between already enumerated dots and still to-be-enumerated dots. Moreover, especially for the decomposition/recomposition strategy, children had to mentally transfer the first enumerated group of dots on the remaining dots to determine the approximate

number of similar groups. If these processes are less efficient in fifth than in seventh graders, then fifth graders would use them less often than young adults.

- 40 Although the present findings of strategy distributions in each age group and for each problem type are consistent with this possibility, directly testing this hypothesis requires analyses of strategy execution. This requires control of strategy use over participants and problem types, so as to have equal number of observations for each Strategy x Configuration x Numerosity condition in each age group. Further studies should investigate this link between strategy use and strategy execution (see Lemaire & Lecacheur, 2002a). The present age-related strategic variations points to the necessity to take into account these variations if we want to understand age-related differences in approximate quantification. Of course, this requires, like here, assessing strategies on each problem to know which strategies children use and how often they use each available strategy, and to manipulate the type of strategies to investigate children's performance independently of strategy use. Only then are we able to provide mechanistic accounts of age-related differences in approximate quantification strategy choices.

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## BIBLIOGRAPHY

- Ansari, D., Donlan, C., & Karmiloff-Smith, A. (2007). Typical and atypical development of visual estimation abilities. *Cortex*, 43(6), 758-768.
- Barrouillet, P., Mignon, M., & Thevenot, C. (2008). Strategies in subtraction problem solving in children. *Journal of Experimental Child Psychology*, 99, 233-251.
- Bevan, W., Maier, R. A., & Helson, H. (1963). The influence of context upon the estimation of number. *American Journal of Psychology*, 76, 464-469.
- Booth, J.L., & Siegler, R.S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology*, 41, 189-201.
- Camos, V. (2003). Counting strategies from 5 years to adulthood: Adaptation to structural features. *European Journal of Psychology of Education*, 18, 251-265.
- Crites, T. (1992). Skilled and less skilled estimators' strategies for estimating discrete quantities. *The Elementary School Journal*, 92(5), 601-619.
- Dehaene, S., & Mehler, J. (1992). Cross-linguistic regularities in the frequency of number words. *Cognition*, 43, 1-29.
- Ericsson, K. A., & Simon, H. A. (1993). *Protocol analysis: Verbal reports as data* (Revised Edition). Cambridge, MA: MIT Press.
- Frith, C. D., & Frith, U. (1972). The solitary illusion: An illusion of numerosity. *Perception & Psychophysics*, 11, 409-410.
- Gandini, D., Lemaire, P., Anton, J.L., & Nazarian, B. (2008a). Neural correlates of approximate quantification strategies in young and older adults: An fMRI study. *Brain Research*, 1246, 144-157.

- Gandini, D., Lemaire, P., & Dufau, S. (2008b). Older and young adults' strategies in approximative quantification. *Acta psychologica*, 129(1), 175-189.
- Ginsburg, N. (1991). Numerosity estimation as a function of stimulus organization. *Perception*, 20, 681-686.
- Hitch, G. (2006). Working memory in children: A cognitive approach. . In E. Bialystok & F. I. M. Craik (Eds.), *Lifespan cognition. Mechanisms of change.* . Oxford: Oxford University Press.
- Kirk, E. P., & Ashcraft, M. H. (2001). The perils and promise of using verbal reports to study math strategies. *Journal of Experimental Psychology: Learning Memory & Cognition*, 27, 157-175.
- Krueger, L. E. (1972). Perceived numerosity. *Perception & Psychophysics*, 11, 5-9.
- Lemaire, P. (2005). Strategic aspects of human cognition: Implications for understanding human reasoning. In M. Roberts (Eds), *Cognitive Strategies and Human Reasoning* (pp. 11-29). London: Psychology Press.
- Lemaire, P., & Lecacheur, M. (2002a). Children' strategies in computational estimation. *Journal of Experimental Child Psychology*, 80(4), 281-304.
- Lemaire, P., & Lecacheur, M. (2002b). Children's strategy use in spelling. *Developmental Science*, 5(1), 43-48.
- Lemaire, P., & Siegler, R. S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology: General*, 124, 83-97.
- Luwel, K., Lemaire, P., & Verschaffel, L. (2005). Children's strategies in numerosity judgment. *Cognitive Development*, 20, 448-471.
- Luwel, K., Verschaffel, L., Onghena, P., & De Corte, E. (2003a). Flexibility in strategy use: Adaptation of numerosity judgment strategies to task characteristics. *European Journal of Psychology of Education*, 15(2), 246-266.
- Luwel, K., Verschaffel, L., Onghena, P., & De Corte, E. (2003b). Strategic aspects of numerosity judgment: the effect of task characteristics. *Experimental Psychology*, 50(1), 63-75.
- Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General*, 111, 1-22.
- Opfer, J. E., & Siegler, R. S. (2007). Representational change and children's numerical estimation. *Cognitive Psychology*, 55, 169-195.
- Robinson, K. (2001). The validity of verbal reports in children's subtraction. *Journal of Educational Psychology*, 93, 211-222.
- Shrager, J., & Siegler, R. S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Science*, 9, 405-410.
- Siegel, A. W., Goldsmith, L. T., & Madson, C. R. (1982). Skill in estimation problems of extent and numerosity. *Journal of Research in Mathematics Education*, 13(3), 211-232.
- Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. New York: Oxford University Press.
- Siegler, R. S. & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, 75, 428-444.
- Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: evidence for multiple representations of numerical quantity. *Psychological Science*, 14, 237-243.



- Simon, T.J. (1997). Reconceptualizing the origins of number knowledge: A « non-numerical » account. *Cognitive Development*, 12, 349-372.
- Smith-Chant, B. L., & LeFevre, J. (2003). Doing as they are told and telling it like it is: Self-reports in mental arithmetic. *Memory & Cognition*, 31, 516-528.
- Trick, L.M., & Pylyshyn, Z.W. (1994). Why are small and large numbers enumerated differently? A limited-capacity preattentive stage in vision. *Psychological Review*, 101, 80-102.
- Vos, P. G., van Oeffelen, M. P., Tibosch, H. J., & Allik, J. (1988). Interactions between area and numerosity. *Psychological Research*, 50, 148-154.
- Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, 74, 1-11.

## NOTES

1. These specific numerosities were chosen as Dehaene and Mehler (1992) have shown that people encountered these very often in their environment. Using well-known numerosities maximized the possibility of observing pure age strategy variations and reduced the possibility of observing age differences in terms of memory representations. Indeed, if we used less-known numerosities, we would run the risk that strategy variations observed between the two age groups were due to differences in availability of memory representations rather than differences in availability of strategies.
  2. This procedure was used because participants often thought aloud while estimating collections of dots. So, recording response time from participants' first vocalization was difficult. When participant gave two responses, the participant's first response was taken into account.
  3. Only participants using more than one strategy were included in these analyses. So, ANOVAs were run on 21 fifth graders and 24 seventh graders.
  4. As in the previous analyses, only participants using more than one strategy were included in these analyses (i.e., 21 fifth graders and 24 seventh graders).
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## ABSTRACTS

Fifth and seventh graders were asked to provide a quick and rough estimate of the number of items in collections of 11–79 items. We collected verbal strategy reports and performance on each item. Results showed that: (a) participants used six different estimation strategies, (b) overall, fifth and seventh graders used the same set of strategies but varied in how often they used each strategy, (c) fifth graders' strategy repertoire was smaller than seventh graders', and (d) strategy selection varied as a function of children's age, and of numerosities and configurations of items. These findings show that different processes are available for approximate quantification in both fifth and seventh graders, and document age-related differences in children's approximate quantification.

Des enfants de 10 et 12 ans ont été testés sur une tâche de quantification approximative, consistant à donner le nombre approximatif de points contenus dans des grilles de 10x10. Nous

avons collecté plusieurs indicateurs comportementaux des performances : les protocoles verbaux, les latences et la précision des réponses. Les résultats ont montré que : (a) les enfants utilisaient six stratégies différentes d'estimation, (b) globalement, les enfants des deux groupes d'âge disposaient du même répertoire stratégique mais se distinguaient sur la fréquence d'utilisation de chacune de ces stratégies, (c) les enfants de 10 ans utilisaient, en moyenne, moins de stratégies que les enfants de 12 ans, et (d) la sélection stratégique variait en fonction de l'âge des enfants et des caractéristiques des configurations (disposition spatiale des items et numérosité). Ces résultats suggèrent que différents processus sont disponibles pour réaliser une tâche de quantification approximative, chez les enfants de 10 et 12 ans. De plus, cette étude permet d'approfondir nos connaissances concernant les différences liées à l'âge dans le domaine de la quantification approximative.

## INDEX

**Mots-clés:** développement cognitif, quantification approximative, stratégie

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